Rydberg Atoms Ionisation by Microwave Field and Electromagnetic Pulses

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Abstract. A simple theory of the Rydberg atoms ionisation by electromagnetic pulses and microwave field is presented. The analysis is based on the scale transformation which reduces the number of parameters and reveals the functional dependencies of the processes. It is shown that the observed ionisation of Rydberg atoms by subpicosecond electromagnetic pulses scale classically. The threshold electric field required to ionise a Rydberg state may be simply evaluated in the photonic basis approach for the quantum dynamics or from the multiphoton ionisation theory.

INTRODUCTION

Highly excited atoms in electromagnetic fields serve as a simple important examples of the strongly driven by an external driving field non-linear systems with the stochastic behaviour and provide an unique opportunity to explore the quantum manifestations of the classical chaos and the quantum-classical correspondence for chaotic systems. That is why great attention has been devoted to the experimental and theoretical investigations of the dynamics of the Rydberg electron in the strong microwave field and to ionisation peculiarities of the Rydberg states (1-4).

Recently the ultrashort, half-cycle electromagnetic pulses with central frequencies around 1 THz have been created in photoconducting switches and used for ionisation of Rydberg atoms (5). One peculiarity of this process is that the duration of the electromagnetic pulse is short compared to the internal time, the Kepler orbital time $T_k = 2\pi n_0^3$, of the atom under study. This results to the specific features in scaling of the threshold field required to ionise a Rydberg state. The classical theory explains the half-cycle pulses (HCP) ionisation only qualitatively (5-7) while quantum corrections in the numerical calculations (6-7) are negligible.

Here we show that (a) the observations (5) *consist with the general classical scaling relations* for the hydrogenic atom in a microwave field (8), (b) the same scaling and close threshold field results from the multiphoton ionisation theory (1), and (c) it may be simply evaluated in the photonic basis approach for the quantum dynamics.

SCALING RELATIONS

The analysis of the classical and quantum dynamics of the hydrogenic atom in a microwave field may be simplified introducing the scale transformation which reduces the number of parameters and reveals the functional dependencies of the processes (8-10).

The Hamiltonian in atomic units a.u. ($e = \hbar = m = 1$) for the hydrogenic atom in a microwave field of frequency ω and field strength F is

$$H = \mathbf{p}^2/2 - 1/r + zF\sin(\omega\tilde{t} + \varphi) \tag{1}$$

where $\mathbf{r}$ and $\mathbf{p}$ are the position and momentum of the electron. Measuring the time of the action of the microwave field on the hydrogenic atom in the field periods one can introduce the scale transformation (8):

$$t = \omega\tilde{t}, \quad \mathbf{r}_s = \omega^{2/3}\mathbf{r}, \quad \mathbf{p}_s = \mathbf{p}/\omega^{1/3}, \quad F_s = F/\omega^{4/3}, \quad H_s = H/\omega^{2/3} \tag{2}$$

where the scaled Hamiltonian is

$$H_s = \mathbf{p_s}^2/2 - 1/r_s + z_s F_s \sin(t + \varphi) \tag{3}$$

The scaled time-dependent Schrödinger equation can be expressed as

$$i\omega^{1/3}\frac{\partial\Psi}{\partial t} = (H_s^0 + V_s)\Psi \tag{4}$$

$$H_s^0 = \frac{\omega^{2/3}}{2}\left[-\frac{1}{r_s^2}\frac{\partial}{\partial r_s}\left(r_s^2\frac{\partial}{\partial r_s}\right) + \frac{l(l+1)}{r_s^2}\right] - \frac{1}{r_s}$$
$$V_s = z_s F_s \sin(t + \varphi) \tag{5}$$

The scaled energy spectrum of the unperturbed hydrogen atom is

$$E_s = -1/2\omega^{2/3}n^2 = -1/2s^{2/3} \tag{6}$$

where $s = \omega/(-2E)^{3/2}$ is the relative field frequency: the ratio of the microwave frequency ω to the Kepler orbital frequency of the electron $\Omega = (-2E)^{3/2}$.

Expressions (3) shows that the classical motion of the electron with the definite initial conditions depends only on the scaled field strength and, on the

contrary, the scaled threshold field strength for the onset of classical chaos or for the ionisation of the Rydberg atom F_s^{th} is a function of the initial scaled energy or initial relative field frequency $s_0 = \omega n_0^3$, i.e. $F_s^{th} = f(s_0)$. The concrete form of the function $f(s_0)$ depends on the ionisation mechanism, which is different for low, intermediant and high relative frequencies s_0. Namely, the static field ($s_0 \to 0$) ionisation threshold is $F_s^{st} \simeq 0.130/s_0^{4/3}$, while the threshold for onset of classical chaos in high frequency ($s_0 \gg 1$) microwave field is at $F_s^{MW} \simeq 1/49s^{5/3}$, Refs. (1-4,8-10). From the finding, Ref. (5), of the threshold field for HCP ionisation, $F^{HCP} \simeq 0.3/n_0^2\tau_{HCP}^{2/3}$ if $s_0 \geq 1$, we have the scaled threshold field $F_s^{HCP} \simeq 0.14/s_0^{2/3}$. A simple classical model explains the observed scaling but the theoretical and experimental ionisation thresholds differ by a factor 2.5, Refs. (5-7, 10). So, *all the three mechanisms of classical ionisation are in the framework of the general classical scaling*, Ref. (8): the scaled threshold field is a function of the relative field frequency, but the concrete form of the function depends on the ionisation mechanism.

According to equations (4) and (5) the motion of the quantum hydrogen atom in a monochromatic field is governed in addition to the scaled field by the scaled Planck constant $\hbar = \omega^{1/3}$ (in a.u.).

QUANTUM IONISATION THEORY

The direct first-order quantum ionisation of Rydberg atom by electromagnetic pulses is relatively weak and exhibits no threshold field dependence since it is proportional to F^2. Therefore, we consider the multistep ionisation processes. For simplification of the problem one can introduce the photonic basis and calculate quantum transitions in the model system with an equidistant energy spectrum, Ref. (8). Using the known expressions for the dipole matrix elements between excited atomis states one can evaluate the ionisation probability by means of Prenyakov and Urnov's model (11) (see also, Refs. (8-10)):

$$P_i \simeq J_{N_i}^2(K) \simeq (2\pi N_i)^{-1}(eK/2N_i)^{2N_i}, \quad K \simeq \pi F_s/2\omega^{1/3}. \qquad (7)$$

Here $J_N(K)$ is the Bessel function, $e \simeq 2.718\ldots$, $N_i \simeq 1/2n_0^2\omega$ is the number of photons required for ionisation and the effective dipole matrix elements for transitions between the photonic states were used. Thus, the ionisation is appreciable if $eK/2N_i \simeq 1$, or $F_s^{Phb} \simeq 2/e\pi s_0^{2/3}$, which is close to the observations, Ref. (5). On the other hand, the rate of the multiphoton ionisation is $\omega_i \propto (7.05n_0^2F/\omega^{2/3})^{2N_i}$, Ref. (1). Therefore, the threshold field for multiphoton ionisation is $F_s^{MPh} \simeq 1/7.05s_0^{2/3}$ which is *in agreement with the observed, Ref. (5), scaling and absolute experimental results.*

CONCLUSIONS

Scale transformations for the dynamics of the hydrogenic atom in an electromagnetic field are wery useful and enables one to maximally simplify the analyses of the dynamic processes and reduce the number of parameters of the problem. The classical motion depends only on the scaled field strength while the quantum dynamics depends, in addition, on the scaled Planck constant.

Simple analytical approach based on the photonic basis yields to the correct description of the quantum dynamics for the hydrogen atom in a microwave field including the quantum suppression of chaotic diffusion effect and enables one to clarify relations between the different quantum-classical correspondence conditions.

The observed scaling in the ionisation of Rydberg atoms by subpicosecond half-cycle electromagnetic pulses consist with the general classical scaling for the hydrogenic atom in a microwave field. The same scaling and close threshold fields for ionisation result from the multiphoton ionisation theory and may be simply evaluated in the photonic basis approach. The observed peculiarity of the threshold field scaling results from the shortness of the electromagnetic pulses.

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